Part 2 4.6 Section: Logarithmic Differentiation

Uses properties of logarithms

I.e. $\ln xy = \ln x + \ln y$

$\ln \frac{x}{y} = \ln x - \ln y$

$\ln x^4 = 4 \cdot \ln x$

} 3 Rules that help make derivatives easier to find.

(#44) $y = (3x+2)(8x-5)$

Step 1 Take log of both sides, so

$\ln y = \ln (3x+2)(8x-5)$

Step 2 Use rules above to get addition problem, so b/c its multiplication use $\ln x + \ln y$

, Thus $\ln y = \ln(3x+2) + \ln(8x-5)$

Step 3 Find derivatives

$$\frac{1}{y} \cdot y' = \frac{1}{3x+2} \cdot 3 + \frac{1}{8x-5} \cdot 8$$

Step 4 Simplify: $\boxed{y' = \left(\frac{3}{3x+2} + \frac{8}{8x-5}\right) y}$

With logarithmic this would be Final answer.

Redo by plugging in y' into equation

, so $y' = \left(\frac{3}{3x+2} + \frac{8}{8x-5}\right)(3x+2)(8x-5)$

$= 3(8x-5) + 8(3x+2)$

$= 24x - 15 + 24x + 16$

$= 48x + 1$

Algebraically working it out:

$y = (3x+2)(8x-5)$

FOILED $y = 24x^2 - 15x + 16x - 10$

$y = 24x^2 + x - 10$

now find y' by simplifying

$\boxed{y' = 48x + 1}$ * Use this to recheck/verify previous answer

Example: $y = \left(\dfrac{(2x-1)^{18}(6x-3)^{12}(5x+1)^3}{(2x-3)^{17}(4x+3)^6}\right)^2$

Step 1 External power - Get rid of it by multiplying it by internal powers, so

$$y = \frac{(2x-1)^{36}(6x-3)^{24}(5x+1)^6}{(2x-3)^{34}(4x+3)^{12}}$$

Step 2 Get powers down by using ln, so

$$\ln y = \ln\left(\frac{(2x-1)^{36}(6x-3)^{24}(5x+1)^6}{(2x-3)^{34}(4x+3)^{12}}\right)$$

Step 3 Use $\ln x + \ln y$ & $\ln x - \ln y$ rules, so

$$\ln y = 36\ln(2x-1) + 24\ln(6x-3) + 6\ln(5x+1) - 34\ln(2x-3) - 12\ln(4x+3)$$

Step 4 Take derivative of each 5 pieces individually

$$\frac{1}{y}\cdot y' = 36\cdot\frac{1}{2x-1}\cdot 2 + \frac{24}{6x-3}\cdot 6 + \frac{6}{5x+1}\cdot 5 - \frac{34}{2x-3}\cdot 2 - \frac{12}{4x+3}\cdot 4$$

Simplified $\left(\dfrac{72}{2x-1} + \dfrac{144}{6x-3} + \dfrac{30}{5x+1} - \dfrac{68}{2x-3} - \dfrac{48}{4x+3}\right)$

Step 5 multiply by (y) to get y' by itself

, so $y' = \left(\dfrac{72}{2x-1} + \dfrac{144}{6x-3} + \dfrac{30}{5x+1} - \dfrac{68}{2x-3} - \dfrac{48}{4x+3}\right) y$